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CHARACTERIZATIONS OF ONE-SIDED FRACTIONAL LÉVY MOTIONS

by

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Technical Report No. 289

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CHARACTERIZATIONS OF ONE-SIDED LINEAR FRACTIONAL LÉVY MOTIONS[*]

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ABSTRACT

We show that the only self-similar stable processes with stationary increments whose left-equivalent (resp. right-equivalent) stationary processes are nonanticipating (resp. fully anticipating) moving averages are the left (resp. right) linear fractional Lévy motions.

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1. Introduction and characterizations

A stochastic process $X = (X(t))_{t\in\mathbb{R}}$ is self-similar with parameter $H > 0$ (H-ss) if $X(c\cdot) \stackrel{d}{=} c^H X(\cdot)$ for all $c > 0$, and has stationary increments (si) if $X(\cdot + b) - X(b) \stackrel{d}{=} X(\cdot)$ for all $b \in \mathbb{R}$, where $\stackrel{d}{=}$ means the equality of all finite-dimensional distributions. Every H-ss si process is stochastically continuous [V1]. A real-valued stochastic process X is α-stable if all finite-dimensional distributions are α-stable where $0 < \alpha \leq 2$, and when $\alpha = 2$, it is Gaussian.

The only H-ss si Gaussian process is the fractional Brownian motion (see [MV]). On the other hand, when $0 < \alpha < 2$, many H-ss si α-stable processes have been recognized in the past ten years (see [KM]). Among these, the linear fractional Lévy motion seems to be the most important example of an H-ss si α-stable process for $0 < \alpha < 2$, and its properties have been well studied (see, e.g., [CM], [ST], [ALT]).

As discussed in our previous paper [CM], to any stochastically continuous si α-stable process $X = (X(t))_{t\in\mathbb{R}}$ with $1 < \alpha \leq 2$ correspond two stationary α-stable processes $Y_L = (Y_L(t))_{t\in\mathbb{R}}$ and $Y_R = (Y_R(t))_{t\in\mathbb{R}}$ in the following way:

$$(1.1)\quad Y_L(t) = \int_{-\infty}^{0} e^{u}[X(t)-X(t+u)]\,du = X(t) - \int_{-\infty}^{t} e^{u-t}\,X(u)\,du,\quad t \in \mathbb{R},$$

and

$$(1.2)\quad Y_R(t) = \int_{0}^{\infty} e^{-u}[X(t)-X(t+u)]\,du = X(t) - \int_{t}^{\infty} e^{t-u}\,X(u)\,du,\quad t \in \mathbb{R}.$$

A straightforward calculation gives for all $s < t$,

$$(1.3)\qquad X(t) - X(s) = Y_L(t) - Y_L(s) + \int_s^t Y_L(u)\,du$$

and

$$(1.4)\qquad X(t) - X(s) = Y_R(t) - Y_R(s) - \int_s^t Y_R(u)\,du.$$

((1.3) is given in (2.2) on p. 307 in [CM], where the minus sign before the integral should be replaced by a plus sign as in (1.3)). (1.4) is derived from (1.2) in the same way as (1.3) is derived from (1.1) in [CM].

The existence of the integrals in (1.1) and (1.2), and the derivation of (1.3) and (1.4) from (1.1) and (1.2), respectively, are justified only when $1 < \alpha \leq 2$, (but both are expected to be valid for $0 < \alpha \leq 1$ as well). Therefore, whenever Y_L or Y_R is considered, we assume $1 < \alpha \leq 2$. The correspondence between X and Y_L is one-to-one and so is the correspondence between X and Y_R. Furthermore, it follows from (1.1) and (1.3) that either one of X and Y_L is expressed linearly in terms of past values of the other, so they are left-equivalent in the sense that if $\mathcal{L}_L(X,t)$ is the closure in probability of all finite linear combinations of the values to the left of t: $\{X(u),\ u \leq t\}$, then

$$\mathcal{L}_L(X,t) = \mathcal{L}_L(Y_L,t), \text{ for any } t \in \mathbb{R}.$$

Similarly it follows from (1.2) and (1.4) that if $\mathcal{L}_R(X,t)$ is the closure in probability of all finite linear combinations of the values to the right of t: $\{X(u),\ u \geq t\}$, then

$$\mathcal{L}_R(X,t) = \mathcal{L}_R(Y_R,t), \text{ for any } t \in \mathbb{R}.$$

We will call Y_L and Y_R the left-equivalent and right-equivalent stationary process of the si process X.

One of the most important classes of stationary stable processes consists of moving averages:

$$(1.5) \qquad Y(t) = \int_{-\infty}^{\infty} g(t-u)\, dM_\alpha(u), \quad t \in \mathbb{R},$$

where $0 < \alpha \le 2$, $g \in L^\alpha(\mathbb{R})$, M_α is α-stable Lévy motion (Brownian motion when $\alpha = 2$), i.e., has stationary independent α-stable increments with

$$(1.6) \quad E[\exp\{i\theta[M_\alpha(t)-M_\alpha(s)]\}] = \begin{cases} \exp\{-|t-s|\theta^2/2\} & \text{if } \alpha = 2, \\ \exp\{-|t-s||\theta|^\alpha[1 - i\beta\ \mathrm{sgn}(\theta)\ \tan(\pi\alpha/2)]\} & \text{if } 0 < \alpha < 2,\ \alpha \ne 1, \\ \exp\{-|t-s||\theta|[1 + i\beta(2/\pi)\mathrm{sgn}(\theta)\ell n|\theta|]\} & \text{if } \alpha = 1, \end{cases}$$

where $|\beta| \le 1$. We note that for each $\alpha \in (0,2]$, M_α has $1/\alpha$-ss increments.

When $g(x) = 0$ for $x < 0$, Y is a nonanticipating moving average process as it is expressed in terms of the past increments of the stable Lévy motion M_α: $Y(t) = \int_{-\infty}^{t} g(t-u)\, dM_\alpha(u)$; and when $g(x) = 0$ for $x > 0$, Y is fully anticipating moving average as it is expressed in terms of the future increments of the stable Lévy motion M_α: $Y(t) = \int_t^\infty g(t-u)\, dM_\alpha(u)$.

The linear fractional Lévy motion $(\Delta_{\alpha,H}(a,b;t))_{t\in\mathbb{R}}$ is an H-ss si α-stable process defined for $0 < H < 1$, $0 < \alpha \le 2$, $H \ne 1/\alpha$, $a,b \in \mathbb{R}$, by

$$(1.7) \qquad \Delta_{\alpha,H}(a,b;t) = \int_{-\infty}^{\infty} \{a[(t-u)_+^{H-1/\alpha} - (-u)_+^{H-1/\alpha}] + b[(t-u)_-^{H-1/\alpha} - (-u)_-^{H-1/\alpha}]\}\, dM_\alpha(u),$$

with the convention $0^\gamma = 0$ even for $\gamma < 0$, where we assume $\beta = 0$ in (1.6) if $\alpha = 1$. When $\alpha = 2$, the processes $\Delta_{2,H}(a,b;\cdot)$ are multiples of the fractional Brownian motion for all a and b. However, when $0 < \alpha < 2$, different lines through the origin of the (a,b) plane give different processes ([CM], [ST]).

We now assume $1 < \alpha < 2$. As shown in [CM], the left-equivalent stationary α-stable process $Y_L(a,b;\cdot)$ of the linear fractional Lévy motion $\Delta_{\alpha,H}(a,b;\cdot)$ is a moving average

(1.8) $$Y_L(a,b;t) = \int_{-\infty}^{\infty} g_{a,b}(t-u)\, dM_\alpha(u), \qquad t \in \mathbb{R},$$

where

(1.9) $$\begin{aligned} g_{a,b}(x) &= a\left[x_+^{H-1/\alpha} - e^{-x}\int_0^{x_+} e^v\, v^{H-1/\alpha}\, dv\right] \\ &\quad + b\left[x_-^{H-1/\alpha} - e^{-x}\int_{x_-}^{\infty} e^{-v}\, v^{H-1/\alpha}\, dv\right] \\ &=: a\, g_1(x) + b\, g_2(x). \end{aligned}$$

When $Y_L(a,b;\cdot)$ is a nonanticipating moving average of M_α, namely $g_{a,b}(x) = 0$ for $x < 0$, then $b = 0$, since for $x < 0$, $g_1(x) = 0$ but $g_2(x) \neq 0$. The linear fractional Lévy motion with $b = 0$, $\Delta_{\alpha,H}(a,0;\cdot)$, is the one introduced in [TW] and is called left linear fractional Lévy motion. Hence, given a linear fractional Lévy motion, if its left-equivalent stationary process Y_L is a nonanticipating moving average, then it is a left linear fractional Lévy motion.

In this paper we generalize this characterization, replacing the class of linear fractional Lévy motions, by the class of all H-ss si α-stable processes whose left-equivalent stationary stable processes are nonanticipating moving averages.

A similar situation prevails for the right-equivalent stationary process Y_R. As in (1.8) and (1.9), the right equivalent stationary α-stable process Y_R of the linear fractional Lévy motion $\Delta_{\alpha,H}$ is a moving average

(1.10) $$Y_R(a,b;t) = \int_{-\infty}^{\infty} h_{a,b}(t-u)\, dM_\alpha(u),$$

where

(1.11)
$$h_{a,b}(x) = a\left[x_+^{H-1/\alpha} - e^x \int_{x_+}^{\infty} e^{-v}\, v^{H-1/\alpha}\, dv\right] + b\left[x_-^{H-1/\alpha} - e^x \int_0^{x_-} e^{v}\, v^{H-1/\alpha}\, dv\right]$$
$$=: a\, h_1(x) + b\, h_2(x).$$

When $Y_R(a,b;\cdot)$ is a fully anticipating moving average of M_α, namely $h_{a,b}(x) = 0$ for $x > 0$, then $a = 0$, since for $x > 0$, $h_2(x) = 0$ but $h_1(x) \neq 0$. Thus, given a linear fractional Lévy motion, if its right-equivalent stationary process Y_R is fully anticipating moving average, then it is a right linear fractional Lévy motion; and we also generalize this characterization to general H-ss si α-stable process as follows.

<u>Theorem 1</u>. Fix $1 < \alpha < 2$, $0 < H < 1$, $H \neq 1/\alpha$. Let X be a nondegenerate H-ss si α-stable process.

(i) If its left-equivalent stationary process Y_L is a nonanticipating moving average, then
$$X(\cdot) \stackrel{d}{=} \Delta_{\alpha,H}(a,0;\cdot) \quad \text{for some } a \neq 0.$$

(ii) If its right-equivalent stationary process Y_R is a fully anticipating moving average, then
$$X(\cdot) \stackrel{d}{=} \Delta_{\alpha,H}(0,b;\cdot) \quad \text{for some } b \neq 0.$$

We suspect that the linear fractional Lévy motions $\Delta_{\alpha,H}$ of (1.7) are the only H-ss si α-stable processes whose left- (or right-) equivalent stationay stable processes are moving averages. However, we are not able to prove this characterization at present.

It is seen from (1.9)-(1.12) that the left-equivalent stationary process Y_L of a nondegenerate linear fractional Lévy motion cannot be a fully

anticipating moving average, because if $g_{a,b}(x) = 0$ for $x > 0$, then $a = b = 0$ and $g_{a,b} \equiv 0$; and, likewise, the right-equivalent stationary process Y_R cannot be a nonanticipating moving average.

A related characterization of the linear fractional Lévy motions would be as the only H-ss si α-stable processes X of the form

$$X(t) = \int_{-\infty}^{\infty} [G(t-u) - G(-u)]\, dM_\alpha(u), \qquad t \in \mathbb{R}, \tag{1.12}$$

where $G(t - \cdot) - G(-\cdot) \in L^\alpha(\mathbb{R})$ for each $t \in \mathbb{R}$. Expressed in this way, this characterization does not involve the corresponding stationary process Y, and hence is stated for all $0 < \alpha < 2$. Vervaat [V2] derives such a characterization under a self-similarity assumption on the kernel of (1.12) of the following form: for all $t, u \in \mathbb{R}$, $c > 0$,

$$G(c(t-u)) - G(-cu) = c^\beta\{G(t-u) - G(-u)\} \tag{1.13}$$

and $G(0) = 0$. Indeed, (1.13) implies immediately that $G(x) = a\, x_+^\beta + b\, x_-^\beta$. This self-similarity of the kernel G also clearly implies the self-similarity of the process X. Theorem 1 (or more precisely, its proof below) establishes this characterization when the self-similarity of G is replaced by that of X, in the case where $G(x) = 0$ for $x < 0$ or $G(x) = 0$ for $x > 0$. We state this characterization in the following.

Theorem 2. Fix $0 < \alpha < 2$, $0 < H < 1$, $H \neq 1/\alpha$. Let X be a nondegenerate H-ss si α-stable process of the form (1.12). If G vanishes on some half line, then X is a one-sided linear fractional Lévy motion.

2. Proof of Theorem 1.

Since the proofs are similar, we only prove (i) of Theorem 1. Also for notational simplicity, we write Y for Y_L and M for M_α.

Since $Y(t) = \int_{-\infty}^{\infty} g(t-u)\, dM(u)$ is a nonanticipating moving average, $g(x) = 0$ for $x < 0$. Since $\int_0^\infty |g(x)|^\alpha\, dx > 0$ (otherwise $Y \equiv 0$ and thus X is degenerate), if we define

$$t_0 = \inf\{t \geq 0 \mid \mathrm{Leb}[x \in (t,t+\delta);\ g(x) \neq 0] > 0 \quad \text{for any } \delta > 0\},$$

then $0 \leq t_0 < \infty$. We then have

$$Y(t) = \int_{-\infty}^{\infty} g(t-u)\, dM(u) = \int_{-\infty}^{\infty} g(t-(v-t_0))\, dM(v-t_0)$$

$$\stackrel{d}{=} \int_{-\infty}^{\infty} g_{t_0}(t-v)\, dM(v),$$

where $g_{t_0}(x) = g(x + t_0)$ and we have used the fact that M has stationary increments. By the definition of t_0,

(2.1) $$\mathrm{Leb}\{x \in (0,\delta);\ g_{t_0}(x) \neq 0\} > 0 \quad \text{for any } \delta > 0,$$

and $g_{t_0}(x) = 0$, $x < 0$. Hereafter we write g for g_{t_0}.

From (1.3) and (1.5), we have for $s < t$,

(2.2) $$X(t) - X(s) \stackrel{d}{=} \int_{-\infty}^{\infty} [g(t-u) - g(s-u) + \int_{s-u}^{t-u} g(v)\, dv]\, dM(u)$$

$$= \int_{-\infty}^{\infty} [G(t-u) - G(s-u)]\, dM(u),$$

where

$$G(x) = g(x) + \int_0^x g(v)\, dv, \quad x \in \mathbb{R}.$$

Note that $G(x) = 0$, $x < 0$, since $g(x) = 0$, $x < 0$.

Fix $t > 0$. Then we have for any $a_1, \ldots, a_N$, $x_1, \ldots, x_N \in \mathbb{R}$ and $c > 0$,

$$\sum_{n=1}^{N} a_n \, [X(c(t + x_n)) - X(cx_n)]$$

$$\stackrel{d}{=} \int_{-\infty}^{\infty} \left\{ \sum_{n=1}^{N} a_n \, [G(c(t + x_n)-u) - G(cx_n - u)] \right\} dM(u)$$

$$= \int_{-\infty}^{\infty} \left\{ \sum_{n=1}^{N} a_n \, [G(c(t + x_n - v)) - G(c(x_n - v)] \right\} dM(cv)$$

$$\stackrel{d}{=} c^{1/\alpha} \int_{-\infty}^{\infty} \left\{ \sum_{n=1}^{N} a_n \, [G(c(t + x_n - v)) - G(c(x_n - v))] \right\} dM(v)$$

(where we have used the $1/\alpha$-self-similarity of M),

$$\sum_{n=1}^{N} a_n \, [X(t + x_n) - X(x_n)]$$

$$\stackrel{d}{=} \int_{-\infty}^{\infty} \left\{ \sum_{n=1}^{N} a_n \, [G(t + x_n - v) - G(x_n - v)] \right\} dM(v).$$

Hence by the self-similarity of X and

$$|E[\exp\{i \int_{-\infty}^{\infty} f(v) \, dM(v)\}]| = \exp\{-\int_{-\infty}^{\infty} |f(v)|^{\alpha} \, dv\},$$

we have for each $c > 0$,

$$c \int_{-\infty}^{\infty} \left| \sum_{n=1}^{N} a_n \, [G(c(t + x_n - v)) - G(c(x_n - v))] \right|^{\alpha} dv$$

$$= c^{\alpha H} \int_{-\infty}^{\infty} \left| \sum_{n=1}^{N} a_n \, [G(t + x_n - v) - G(x_n - v)] \right|^{\alpha} dv,$$

namely, the functions $c^{1/\alpha-H}[G(c(t-\cdot)) - G(c(-\cdot))]$ and $G(t-\cdot) - G(-\cdot)$ have equal L_α-norms of all linear combinations of their translates. Then, by Kanter's result [K], for each fixed $c > 0$,

(2.3) $c^{1/\alpha-H}[G(c(t-u)) - G(-cu)] = \epsilon_c[G(t-u-\tau_c) - G(-u-\tau_c)]$ for a.a. u,

where $\epsilon_c \in \{-1,1\}$, $\tau_c \in \mathbb{R}$, and they may depend on c. We shall show that $\epsilon_c = 1$ and $\tau_c = 0$ for all $c > 0$.

Suppose $\tau_c > 0$. Then *for* u satisfying $\max\{0, t-\tau_c\} < u < t$, (2.3) reduces to

$$c^{1/\alpha-H}\, G(c(t-u)) = 0, \quad \text{a.a. } u,$$

since $G(x) = 0$, $x < 0$. Namely,

$$G(x) = 0 \quad \text{for} \quad \text{a.a.} \quad x \in (0, \min\{ct, c\tau_c\}) =: I_c,$$

and thus

$$g(x) = \int_0^x g(v)\, dv \quad \text{for} \quad \text{a.a.} \quad x \in I_c.$$

The function $f(x) := \int_0^x g(v)\, dv$ is continuous and satisfies $f(x) = \int_0^x f(v)\, dv$ for any $x \in I_c$. Thus $f \equiv 0$ on I_c, implying $g = 0$ a.e. on I_c, which contradicts (2.1).

When we suppose $\tau_c < 0$, it is enough to take u satisfying $\max\{t, -\tau_c\} < u < t - \tau_c$ in order to get a contradiction. Namely, for such u, we have $G(t-u-\tau_c) = 0$, so that $G(x) = 0$ for a.a. $x \in (0, \min\{t, -\tau_c\})$. Thus $\tau_c = 0$ for any $c > 0$, and we obtain

$$c^{1/\alpha-H}[G(c(t-u)) - G(-cu)] = \epsilon_c[G(t-u) - G(-u)] \quad \text{for a.a. } u.$$

We thus have for a.a. $u \in (0,t)$, $c^{1/\alpha-H}\, G(c(t-u)) = \epsilon_c\, G(t-u)$, namely

$$G(cx) = \epsilon_c\, c^{H-1/\alpha}\, G(x) \quad \text{for} \quad \text{a.a.} \quad x \in (0,t). \tag{2.4}$$

Choose $0 < a < b < t$ and integrate (2.4); then

$$\int_a^b G(cx)\, dx = \epsilon_c\, c^{H-1/\alpha} \int_a^b G(x)\, dx$$

so that

$$\int_{ca}^{cb} G(x)\, dx = \epsilon_c\, c^{H-1/\alpha+1} \int_a^b G(x)\, dx. \tag{2.5}$$

If $\int_a^b G(x)\,dx = 0$ for any $0 < a < b < t$, then $G(x) = 0$ a.e. on $(0,ct)$. We thus have the same contradiction as before for each $c > 0$. If $\int_a^b G(x)\,dx \neq 0$ for some $0 < a < b < t$, then we see from (2.5) that ϵ_c is a continuous function of $c > 0$ and thus $\epsilon_c = \epsilon_1 = +1$ for all $c > 0$.

Therefore, we obtain

$$(2.6) \qquad G(cx) = c^{H-1/\alpha} G(x) \quad \text{for all } c > 0 \text{ and a.a. } x \in (0,t).$$

By the same reasoning as before, we see that $G(x_0) \neq 0$ for some $x_0 > 0$ independent of $c > 0$. We thus have from (2.6),

$$(2.7) \qquad G(x) = (x/x_0)^{H-1/\alpha}\, G(x_0) = a\, x^{H-1/\alpha}, \quad \text{for all } x > 0,$$

where $a = G(x_0)\, x_0^{1/\alpha-H} \neq 0$. We know that $G(x) = 0$ for all $x < 0$, and assuming $G(0) = 0$ does not change the distribution of X. So, we have

$$(2.8) \qquad G(x) = a\, x_+^{H-1/\alpha}, \quad \text{for all } x \in \mathbb{R}.$$

Noting that any H-ss si process with $H > 0$ satisfies $X(0) = 0$ a.s. (see, e.g., [V1]), we conclude from (2.2) and (2.8) that

$$X(t) = a \int_{-\infty}^{\infty} [(t-u)_+^{H-1/\alpha} - (-u)_+^{H-1/\alpha}]\, dM(u),$$

which completes the proof.

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